# The generalized second law of thermodynamics in the accelerating universe

Jia Zhou, Bin Wang*

*Department of Physics, Fudan University, 200433 Shanghai*

Yungui Gong†

*School of Physical Science and Technology,*

*Southwest University, Chongqing 400715, China and*

*CASPER, Department of Physics, Baylor University, Waco, TX 76798, USA*

Elcio Abdalla‡

*Instituto de Fisica, Universidade de Sao Paulo,*

*C.P.66.318, CEP 05315-970, Sao Paulo*

## Abstract

We show that in the accelerating universe the generalized second law of thermodynamics holds only in the case where the enveloping surface is the apparent horizon, but not in the case of the event horizon. The present analysis relies on the most recent SNe Ia events, being model independent. Our study might suggest that event horizon is not a physical boundary from the point of view of thermodynamics.

PACS numbers: 98.80.Cq;98.80.-k

*Electronic address: wangb@fudan.edu.cn

†Electronic address: yungui˙gong@baylor.edu

‡Electronic address: eabdalla@fma.if.usp.br

Today there is an overwhelming amount of data showing that our universe is experiencing an accelerated expansion driven by a so called "dark energy". The nature of such previously unforeseen energy still remains a complete mystery, except for the fact that it has negative pressure. In this new conceptual set up, one of the important questions concerns the thermodynamical behavior of the accelerated expanding universe driven by dark energy. It is interesting to ask whether thermodynamics in the accelerating universe can tell us some properties of dark energy. There is at least a possibility to answer this question: it has been argued that there is a deep connection between thermodynamics and gravity after the discovery of the relation between thermodynamics and gravity in black hole physics in 1970's [1]. Thoughts on the profound relation between thermodynamics and gravity have been inspired by general situations, including the cosmological context [2]. A close connection was disclosed between thermodynamics and Friedmann equation derived in Einstein gravity in the cosmological cases [3]. This connection implies that the thermodynamical properties can help understand the dark energy, which gives strong motivation to study thermodynamics in the accelerating universe.

The thermodynamics of the de Sitter universe was considered some years ago [4]. The de Sitter universe experiences accelerated expansion and has only one cosmological horizon analogous to the black hole horizon. It was found that the first law of thermodynamics is valid at this horizon. If the accelerating universe is driven by dark energy with equation of state $w \neq -1$, the single horizon in the de Sitter case will be separated into the apparent horizon and the event horizon. The apparent Hubble horizon of the universe always exists while the event horizon does not. The thermodynamical properties associated with the apparent horizon have been found in a quasi-de Sitter geometry of inflationary universe [5]. In the late time accelerating universe, it was disclosed that the first law of thermodynamics holds in the physically relevant part of the universe enveloped by the dynamical apparent horizon, while does not hold appropriately in the region enveloped by the cosmological event horizon[6]. This result is supported if one considers the relation between thermodynamics and gravity [3].

Besides the first law of thermodynamics, a lot of attention has been paid to the generalized second law of thermodynamics in the accelerating universe driven by dark energy [7, 9, 10]. The generalized second law of thermodynamics is as important as the first law, governing the development of the nature. Using a specific model of dark energy, the generalized second

law as defined in the region enveloped by the apparent horizon as well as in the event horizon was examined in [6]. It was found that it is obeyed in the case of the universe enveloped by the apparent horizon, not otherwise.

In this paper we shall extend our study on the generalized second law of thermodynamics to a general accelerating universe with model independent dark energy equation of state. We use the most recent type Ia supernovae observations to carry out the model independent analysis of the generalized second law of thermodynamics. Recently, the SNe Ia data have been used to examine the energy conditions' violation in the context of the standard cosmology [11, 12].

Within the framework of the standard FRW cosmology,

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\sigma^2) \quad , \tag{1}$$

the evolution of the universe is governed by Friedmann equations

$$H^2 = \frac{8\pi}{3}\rho \quad , \tag{2}$$

$$\dot{H} = -4\pi(\rho + P) \quad , \tag{3}$$

where $H = \dot{a}/a$ is the Hubble parameter. Assuming a perfect cosmological fluid, we have

$$\dot{\rho} + 3H(\rho + P) = 0 \quad , \tag{4}$$

where $\rho, P$ are respectively the energy density and the pressure of the total content of the universe.

The entropy of the universe inside the horizon can be related to its energy and pressure in the horizon by the Gibbs equation [7]

$$TdS_{in} = d(\rho V) + PdV = Vd\rho + (\rho + P)dV. \tag{5}$$

For the dynamical apparent horizon $R_A = 1/H$. Considering the total volume $V = \frac{4\pi}{3}R_A^3$, we have

$$TdS_{in} = (H^{-2} + \dot{H}H^{-4})dH. \tag{6}$$

We assume that the temperature scales as the temperature of the horizon, which we assume to be $T_H = H/(2\pi)$. It is natural to suppose that the temperature of the perfect fluid inside the apparent horizon is $T = bT_H$, where $b$ is a real proportional constant to be figured

out[7, 8]. We limit ourselves to the assumption of the local equilibrium hypothesis that the energy would not spontaneously flow between the horizon and the fluid, the latter would be at variance with the FRW geometry.

In addition to the entropy of the universe inside the apparent horizon, there is a horizon entropy associated to the apparent horizon,

$$S_A = \pi R_A^2 = \pi H^{-2} \quad . \tag{7}$$

In order to check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy $S_{in} + S_A$, which is in the form

$$\dot{S}_{in} + \dot{S}_A = 2\pi H^{-5} \dot{H}[(1-b)H^2 + \dot{H}]b^{-1}. \tag{8}$$

This expression holds in the local equilibrium hypothesis[20]. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. For the non-equilibrium case, an interaction term should be added on the right-hand-side of the above equation which will make the discussion more complicated[21]. Here we limit our discussion on the local equilibrium assumption.

For $b \leq 1$, the generalized second law can be secured provided that

$$\dot{H} \leq (b-1)H^2 \quad \text{or} \quad \dot{H} \geq 0 \quad . \tag{9}$$

However, in the range $(b-1)H^2 < \dot{H} < 0$, the generalized second law breaks down. The first integral of the above equation leads to

$$\dot{a} \geq H_0 a_0^{-b+1} a^b \quad \text{or} \quad \dot{a} \leq H_0 a \quad . \tag{10}$$

Using definitions for the radial coordinate distance, the luminosity distance and distance modulus,

$$r(a) = \int_0^{r(a)} dr = \int_a^{a_0} \frac{da'}{\dot{a}'a'} = \int_a^{a_0} \frac{da'}{Ha'^2} \quad , \tag{11}$$

$$d_L(z) = a_0(1+z)r(a) \quad , \tag{12}$$

$$\mu(z) := m(z) - M = 5 \lg d_L(z) + 25 \quad , \tag{13}$$

we obtain the upper or lower bound on the radial distance necessary to enforce the generalized second law as

$$r(z) \leq \frac{(1+z)^b - 1}{H_0 a_0 b} \quad \text{or} \quad r(z) \geq \frac{z}{H_0 a_0} \quad , \tag{14}$$

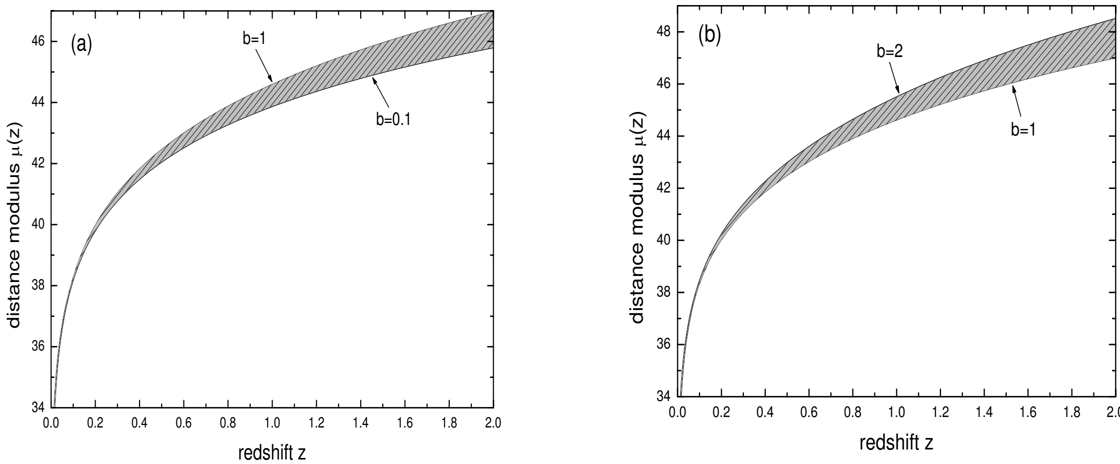


FIG. 1: (a) shows the condition to protect the generalized second law for $b \leq 1$ and (b) shows the condition for $b \geq 1$. We use the central value for the Hubble parameter, $H_0 = 72km{\cdot}s^{-1}{\cdot}Mpc^{-1}$ [13].

where $a_0/a = 1 + z$. Combining (10), (12), (13) and (14), we obtain inequalities for the distance modulus

$$\mu(z) \leq 5\lg\frac{(1+z)[(1+z)^b - 1]}{H_0 b} + 25 \quad \text{or} \quad \mu(z) \geq 5\lg\frac{(1+z)z}{H_0} + 25 \quad . \tag{15}$$

The bounds on the distance modulus as a function of the redshift are shown in Fig.1a, where in the shadow the generalized second law breaks down.

For $b \geq 1$, the generalized second law holds if

$$\dot{H} \geq (b-1)H^2 \quad \text{or} \quad \dot{H} \leq 0 \quad . \tag{16}$$

From these constraints we find that the generalized second law cannot hold when $0 < \dot{H} < (b-1)H^2$. The bounds for enforcing the generalized second law on the distance modulus are

$$\mu(z) \geq 5\lg\frac{(1+z)[(1+z)^b - 1]}{H_0 b} + 25 \quad \text{or} \quad \mu(z) \leq 5\lg\frac{(1+z)z}{H_0} + 25 \quad , \tag{17}$$

which are shown in Fig.1b. The area in shadow violates the generalized second law.

The shadows in Fig.1 shrink when $b \to 1$ and disappear when $b = 1$. The generalized second law of thermodynamics is one of the most general laws governing the nature. Requiring validity of the generalized second law leads to the limit $b \to 1$. This requires that

the temperature of the fluid inside the universe should be in equilibrium with the apparent horizon temperature. The fluid and the horizon must interact for some length of time but finally they must attain thermal equilibrium[7]. In the thermal equilibrium, the generalized second law holds for any value of $\dot{H}$.

When the universe is in equilibrium with the apparent horizon temperature, the line for the distance modulus corresponds to $\dot{H} = 0$, which shows that the total entropy of the universe keeps unchanged. If the observational data accumulate on this line, we will learn that the universe is the de Sitter space with $w = -1$.

In order to shed some light on the generalized second law in the universe from the observational side, it is important to confront the condition of protecting the generalized second law with the current observational data. In this regard, we use the most recent SNe Ia gold sample compiled in[14]. This sample consists of 182 data, in which 16 points with $0.46 < z < 1.39$ were discovered by HST, 47 points with $0.25 < z < 0.96$ from the first year SNLS and the left 119 old data. Results are shown in Fig.2 where we take $b = 1$. Panels (2a) and (2b) show the condition for the generalized second law and data points in the small redshift $z < 0.1$ and $0.1 \leq z \leq 0.27$ respectively, while panel (2c) shows the curve and data for redshift interval $0.27 \leq z \leq 2$.

The dashed line was plotted by choosing $H_0 = 72Kms^{-1}Mpc^{-1}$, which is the central value of different observations for the present Hubble parameter and this value was also taken in plotting the distance modulus in [11]. It is interesting to see that the data follow the line, but are not exactly on the line. This tells us that $\dot{H}$ is not always zero so that our universe is not exactly de Sitter's. For the small redshift $z \leq 0.27$, the majority of SNe events are above the dashed line, while for the bigger redshift $0.27 \leq z \leq 2$, the majority of the SNe events are below the dashed line. For the low redshift ($z \leq 0.27$), the SNe events above the dashed line do not exactly lead $\dot{H} > 0$. This is because we cannot exactly obtain (9) from (15) or (16) from (17). The determination of when $\dot{H} > 0$ is important, since this can tell us the moment when the universe starts to super-accelerate. However the integrated over results in (15)(17) hide the information of $H$. To make clear the behavior of $\dot{H}$ for the SNe data in the low redshift, we can define $f(z) = r(z) - \frac{z}{H_0 a_0}$, where $\frac{z}{H_0 a_0}$ is the value of $r(z)$ when $\dot{H} = 0$. Its first derivative with respect to $z$ reads $f'(z) = \frac{dr}{dz} - \frac{1}{H_0 a_0}$. At $z = 0, f(0) = 0$ and $f'(0) = 0$. In the vicinity of $z = 0, f(0^+) > 0$, so that $f'(0^+) = \lim_{\varepsilon \to 0^+} \frac{f(\varepsilon) - f(0)}{\varepsilon} > 0$ and $f''(0^+) = \lim_{\varepsilon \to 0^+} \frac{f'(\varepsilon) - f'(0)}{\varepsilon} > 0$. The second derivative of $f(z)$ with respect to $z$ has the

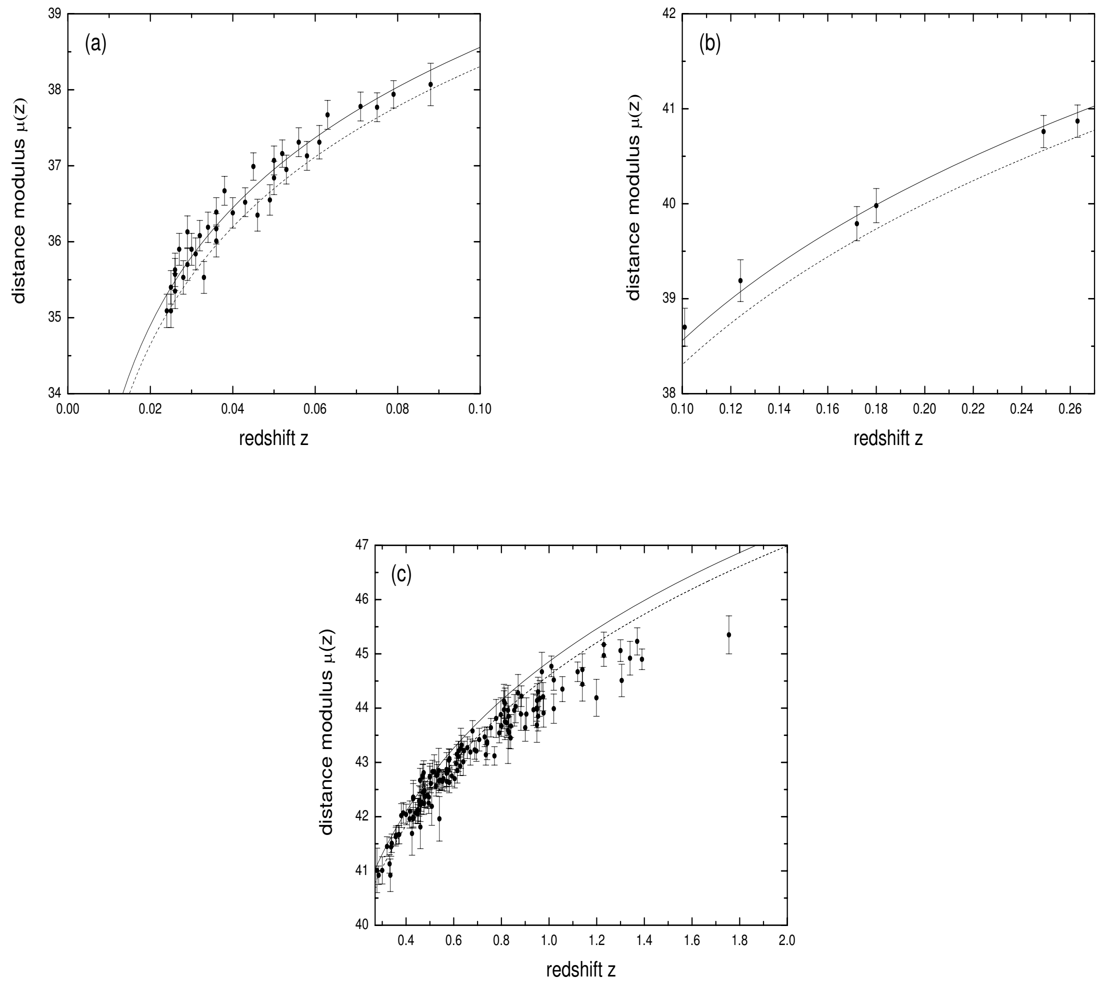


FIG. 2: Panels (2a) and (2b) show the condition for the generalized second law and data points in the small redshift $z < 0.1$ and $0.1 \leq z \leq 0.27$ respectively, while panel (2c) shows the curve and data for redshift interval $0.27 \leq z \leq 2$. The dashed line was plotted by using $H_0 = 72km{\cdot}s^{-1}{\cdot}Mpc^{-1}$, while the solid line is for $H_0 = 64.04km{\cdot}s^{-1}{\cdot}Mpc^{-1}$.

form $f''(z) = \frac{d^2r}{dz^2} = \frac{a\dot{H}}{a_0H^3}$. It's easy to see that $f''(z) > 0$ corresponds to $\dot{H} > 0$ and in this period $f(z)$ experiences a fast increase with $z$. When $\dot{H} = 0$, $f'(z)$ reaches its maximum. For $\dot{H} < 0$, $f(z)$ keeps positive and slowly increases with $z$. The position where $f'(z)$ reaches the maximum (for $\dot{H} = 0$) can be read from the data since $\frac{dr}{dz} = \frac{10^{\frac{\mu-25}{5}}}{a_0(1+z)}(\frac{\ln 10}{5}\frac{d\mu}{dz} - \frac{1}{1+z})$. Using the finite difference method, we show approximate values of $\frac{\Delta\mu_0}{\Delta z}$ and $a_0\frac{\Delta r}{\Delta z}$ by employing the central values of distance modulus of SNe events in the table below. It is clear that there is a maximum value of $\frac{\Delta\mu}{\Delta z}$ around $z_t \sim 0.101$, which corresponds to the biggest jump of

$\Delta r/\Delta z$ where $f'(z)$ reaches its maximum value. We learn that when $z < z_t, \dot{H} > 0$, while $\dot{H} < 0$ for $z > z_t$. Thus the SNe data indicate that $z_t$ is a turning point indicating the transition to the super-acceleration at this redshift. This is consistent with the phantom divide position found in [15]. We expect more accurate SNe data to make the transition position to be determined more exactly. If just from the data listed below, it is also possible that the universe evolves with the oscillating equation of state below and above $-1$, since there are peaks of $\Delta\mu/\Delta z$ and $\Delta r/\Delta z$ around $z = 0.101, 0.180$.

Some observational data above the line $\dot{H} = 0$

| $z$ | 0.088 | 0.101 | 0.124 | 0.172 | 0.180 | 0.249 | 0.263 |
|---|---|---|---|---|---|---|---|
| $\mu_0$ | 38.07 | 38.70 | 39.19 | 39.79 | 39.98 | 40.76 | 40.87 |
| $\frac{\Delta\mu_0}{\Delta z}$ | 34.55 | 48.46 | 21.30 | 23.75 | 23.75 | 11.30 | 7.86 |
| $a_0\frac{\Delta r}{\Delta z}$ | 5665.31 | 10685.55 | 5464.70 | 7811.00 | 8472.30 | 5002.72 | 3342.42 |

In plotting the distance modulus, it was argued that the zero point is arbitrary and a proper value needs to be subtracted from the distance modulus if we use $H_0 = 72 Kms^{-1}Mpc^{-1}$[14]. The same effect to overcome the arbitrariness can be achieved by getting the $H_0$ from the nearby SNe data $z \leq 0.1$ using $d_L(z) = H_0 z$ and it was found that $H_0 = 64.04 Kms^{-1}Mpc^{-1}$[12]. Using $H_0 = 64.04 Kms^{-1}Mpc^{-1}$, in fig.2 we plotted the solid line and we see that nearly the same amount of SNe events are above and below the solid line for $z < 0.1$. We see that the normalization makes it even more difficult to disclose the super-acceleration behavior of the universe from the integrated effect.

Recently Simon et al [18] have published Hubble function $H(z)$ data extracted from differential ages of passively evolving galaxies. The Hubble function is not integrated over, which contains some fine structures covered by the distance modulus. It was observed that $H(z)$ decreases with respect to the redshift $z$ around $z \sim 0.3$ and $z \sim 1.5$ [19], which disclosed the moment when the universe may enter super-acceleration with the total fluid behaves like phantom. The oscillation feature of the equation of state was also observed in studying the Hubble parameter data.

Now we extend our discussion to the universe enveloped by the cosmological event horizon with the volume $V = \frac{4\pi}{3} R_E^3$, where $R_E$ is the event horizon, defined by $R_E = a(t) \int_t^{t_{\max}} \frac{dt'}{a(t')}$.

From the Gibbs law, we find that the entropy inside the event horizon is expressed as

$$TdS_{in} = \frac{4\pi}{3}R_E^3\dot{\rho}dt - R_E^2\dot{H}(HR_E - 1)dt = \dot{H}R_E^2 dt \quad . \tag{18}$$

Including the event horizon entropy $S_E = 2\pi R_E^2$ and supposing $T = b/(2\pi R_E)$, the change of the total entropy has the form

$$\dot{S}_{in} + \dot{S}_E = 2\pi R_E(b^{-1}R_E^2\dot{H} + \dot{R}_E) \quad . \tag{19}$$

in the local equilibrium hypothesis. In order to keep the generalized second law of thermodynamics, we require the above equation to be non-negative, which leads

$$H_0R_E - HR_E \geq b\frac{R_E}{R_{E_0}} - b, \tag{20}$$

where $R_{E_0} = a(t_0)\int_{t_0}^{t_{\max}}\frac{dt}{da}$ is the current value of the future event horizon. Using the definition of the radial distance, we have $R_E = ar + \frac{a}{a_0}R_{E_0}$, and the above equation can be written as

$$H_0ar' + a^{-1}r'\frac{da}{dr'} \geq \frac{b}{R_{E_0}}ar' - b, \tag{21}$$

where $r' = r + \frac{R_{E_0}}{a_0}$. Considering $\frac{da}{dr'} = -\frac{a^2}{a_0}\frac{dz}{dr'}$ and taking $x = a_0r'$, one gets

$$\frac{dz}{dx} - \frac{b(1+z)}{x} \leq H_0 - bR_{E_0}^{-1} \quad . \tag{22}$$

Simply choosing the proportional constant $b$ as unity which means that the perfect fluid is in thermal equilibrium with the event horizon, the generalized second law then requires

$$\frac{dz}{dx} - \frac{1+z}{x} = B, \qquad B \leq H_0 - R_{E_0}^{-1} \quad . \tag{23}$$

Thus, we have

$$z = Ax + Bx\ln x - 1 \quad . \tag{24}$$

Taking account of the present values $z_0 = 0, x_0 = R_{E_0}$, $A = x_0^{-1} - B\ln x_0$ we have

$$z = y + Bx_0y\ln y - 1 \quad , \tag{25}$$

where $y = x/x_0 = (a_0r + R_{E0})/R_{E0}$. This equation shows the relation between the redshift and the radial distance, from which we can learn the behavior of the distance modulus for different $z$. The generalized second law of thermodynamics requires $Bx_0 \leq H_0x_0 - 1$.

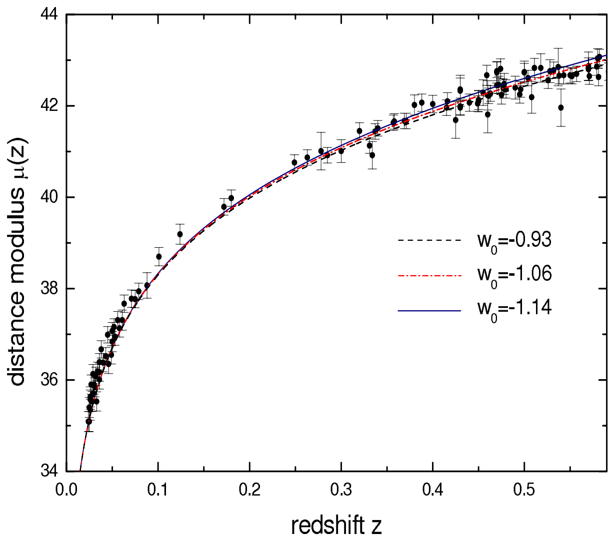


FIG. 3: The distance modulus with $Bx_0 = H_0x_0 - 1$ when the equation of state $w$ is taken to be $-0.93$, $-1.06$, and $-1.14$ respectively. New SNe gold sample data are included. It indicates that the condition for the generalized second law seems to violate by a considerable number of nearby SNe Ia.

Therefore the generalized second law is satisfied in the region above the line of the distance modulus with $Bx_0 = H_0x_0 - 1$.

For the constant equation of state, defining $\epsilon = 3(1+w)/2$, from the Friedmann equation we have $a(t) = t^{1/\epsilon}$ and $R_E = -\epsilon t/(\epsilon - 1)$ and $R_A = \epsilon t$ [6, 16]. For $0 < \epsilon < 1$, $-1 < w < -1/3$, the universe is in the accelerating quintessence-like space, while for $\epsilon < 0$, $w < -1$, the universe is in phantom phase. The apparent horizon and the event horizon do not differ much, they are related by $R_A/R_E = 1 - \epsilon$. Thus $H_0x_0 - 1 = R_{E0}/R_{A0} - 1 = -3(1 + w_0)/(1 + 3w_0)$. For different values of the present equation of state $w_0$, in Fig.3, we plotted the distance modulus as a function of the redshift by taking $H_0 = 72Kms^{-1}Mpc^{-1}$. Within the observational range of the current equation of state $w_0 = -1.06^{+0.13}_{-0.08}$[17], most of the data lie below the lines, which are in violation with the generalized second law requirement. For more negative equation of state or choosing the appropriate normalization of the zero point in the distance modulus, more SN data will be in the region in violation with the generalized second law. This problem cannot be overcome even when we set $w_0 = -0.34$. The observation tells us that the universe enveloped by the event horizon cannot satisfy the generalized second law. This result is general, which supports previous study with the interacting dark energy model[6].

In summary, by extending previous results for the interacting holographic dark energy model [6] to the general dark energy cases, we have examined the generalized second law of thermodynamics in the accelerating universe. We have also confronted the requirement of the generalized second law with the new gold sample 182 SNe observed events. On general grounds, our analysis indicates that the apparent horizon is a good thermal boundary. The accelerating universe enveloped by the apparent horizon satisfies the generalized second law. The SNe events indicate that there is a transition of the equation of state at low redshift, much clearer result could be obtained from the more precise Hubble function data in the future which is not integrated over . However the accelerating universe inside the event horizon does not satisfy the generalized second law. The event horizon in the accelerating universe might not be a physical boundary from the thermodynamical point of view. This result is supported by the new SNe Ia gold samples.

So far our discussion is limited in the local equilibrium hypothesis. Further consideration is called for provided that the horizon and fluid are not in a thermal equilibrium situation.

**Acknowledgments**

This work was partially supported by NNSF of China, Ministry of Education of China and Shanghai Education Commission. E. Abdalla's work was partially supported by FAPESP and CNPq, Brazil. Y.G. Gong was supported by Baylor University, NNSFC under grant No.s 10447008 and 10605042, and SRF for ROCS, State Education Ministry.

[1] J. Bekenstein *Phys. Rev.* **D7** (1973) 2333; S. Hawking *Commun. Math. Phys.* **43** (1975) 199.

[2] T. Jacobson *Phys. Rev. Lett.* **75** (1995) 1260; C. Eling, R. Guedens, and T. Jacobson,*Phys. Rev. Lett.* **96**, 121301 (2006); T. Padmanabhan, *Class. Quantum Grav.* **19**, 5387 (2002).

[3] R.G. Cai and S.P. Kim, *J. High Energy Phys.* **02**, 050 (2005); R.G. Cai and L.M. Cao, *Phys. Rev.* **D75**, 064008 (2007); R.G. Cai and L.M. Cao, hep-th/0612144; M. Akbar and R.G. Cai, gr-qc/0612089; A. Sheykhi, B. Wang and R.G. Cai, hep-th/0701198; A. Sheykhi, B. Wang and R.G. Cai, hep-th/0701261; Yungui Gong, Anzhong Wang, arXiv:0704.0793.

[4] G. W. Gibbons and S. W. Hawking, *Phys. Rev.* **D15** (1977) 2738.

[5] A. Frolov and L. Kofman, *JCAP* **0305** (2003) 009.

[6] Bin Wang, Yungui Gong, Elcio Abdalla, *Phys.Rev.* **D74** (2006) 083520.

[7] G. Izquierdo and D. Pavon, *Phys.Lett.* **B633** (2006) 420

[8] H. Mohseni Sadjadi, *Phys. Lett.* **B645** (2007) 108.

[9] German Izquierdo, Diego Pavon, *Phys.Lett.* B**639** (2006) 1; M.R. Setare, *JCAP* **0701**:023,2007; H. Mohseni Sadjadi, *Phys.Lett.* **B645**,108,2007.

[10] Yungui Gong, Bin Wang, Anzhong Wang, gr-qc/0611155; Yungui Gong, Bin Wang, Anzhong Wang, *JCAP* **0701** (2007) 024.

[11] J. Santos, J.S. Alcaniz, N. Pires, M.J. Reboucas, astro-ph/0702728.

[12] Yungui Gong, Anzhong Wang, Qiang Wu, Yuan-Zhong Zhang, astro-ph/0703583.

[13] W. L. Freedman et al., *Astrophys. J.* **553**, 47 (2001)

[14] A.G. Riess et al. (High-z Supernova Search Team), astro-ph/0611572.

[15] U. Alam, V. Sahni and A. A. Starobinsky, *JCAP* **0406** (2004) 008; D. Huterer and A. Cooray, *Phys. Rev.* **D71** (2005) 023506; Y. Wang and M. Tegmark, astro-ph/0501351.

[16] R. Bousso, *Phys. Rev.* **D71** (2005) 064024.

[17] D. N. Spergel et al, astro-ph/0603449.

[18] J. Simon, L. Verde and R. Jimenez, Phys. Rev. D 71, 123001(2005).

[19] H. Wei, S. N. Zhang, *Phys.Lett.* **B644** (2007) 7; H. S. Zhang, Z. H. Zhu, astro-ph/0703245.

[20] S. R. de Groot and P. Mazur, *Non-equilibrium thermodynamics* (Amsterdam : North-Holland Pub. Co.; New York : Interscience)

[21] D. Jou, J. Casas-Vazquez, and G. Lebon, *Extended Irreversible Thermodynamics* (Berlin ; New York : Springer, 1996.)